Reliability Fundamentals and Analysis Lessons Learned

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Overview

- Reliability Fundamentals
  - Definitions
  - Graphs of a failure process
  - Censored Vs. uncensored data
- Lessons learned from reliability analysis
  - Case Study #1: F-22 infant mortality
  - Case Study #2: F-22 weapon system reliability
  - Case Study #3: RQ-1 Predator failure analysis

Reliability Fundamentals

Definitions

- Statistical methods are used to determine part reliability using part failure data
- **Failure -** The inability of a component or system to perform its intended function for a specified time under specified environment conditions
- **Reliability -** The probability that a component or system will perform its required function satisfactorily for a specified *period* of time when used under stated operating conditions
- **Maintainability -** The probability that a failed component or system will be restored or repaired to a specified condition within a *period* of time when maintenance is performed in accordance with pre-specified procedures
- **Availability -** The probability that a component or system is performing its required function at a given *point* in time when used under stated operating conditions

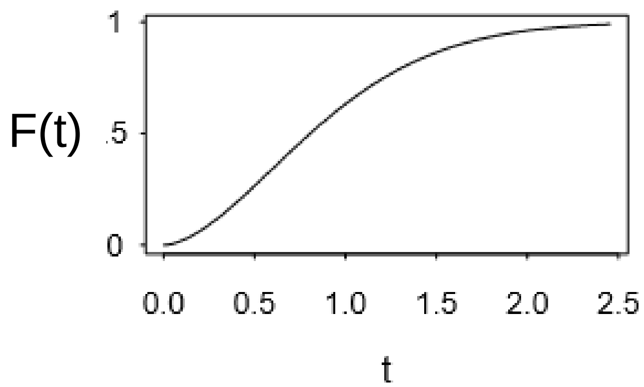
A Word About "Time"

- Unit of measurement for "time"
  - Hours, minutes, seconds
  - Cycles
  - Miles
  - Rounds
  - Etc.
- Whatever "life units" cause the system to age
  - Thermal cycles
  - Take offs and landings
  - Flight time

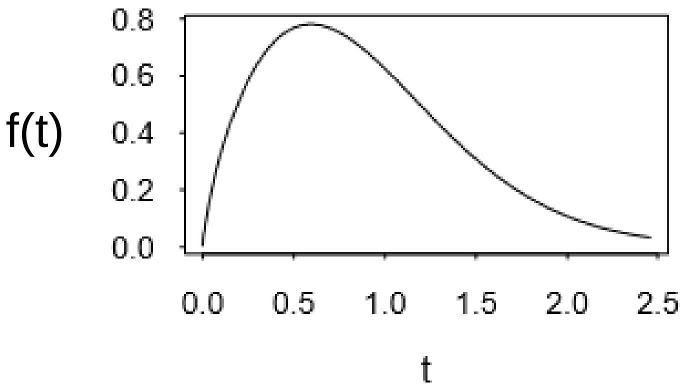
Graphs of a Failure Process

- A failure process, represented by a random variable *T* (time to failure) may be uniquely characterized by any of the following four functions:

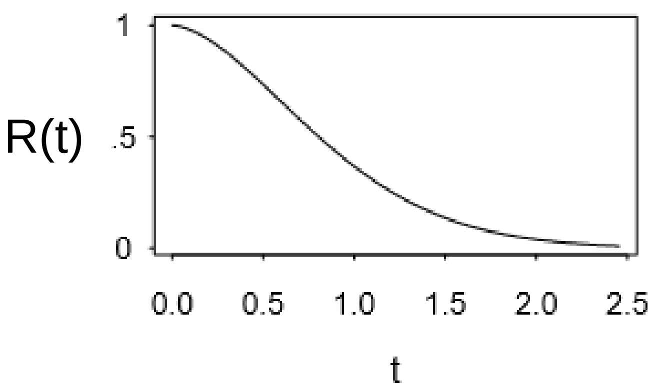
Cumulative Distribution Function



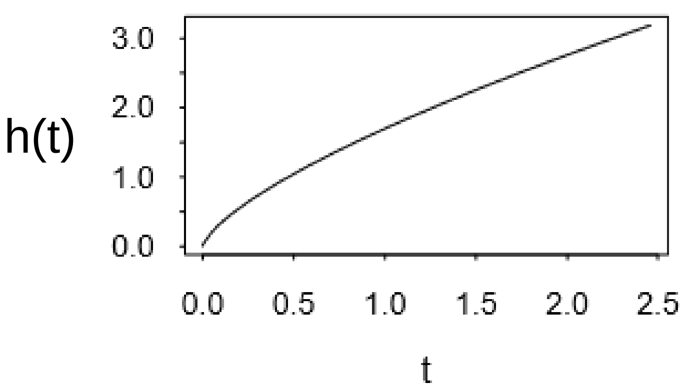
Probability Density Function



Reliability Function



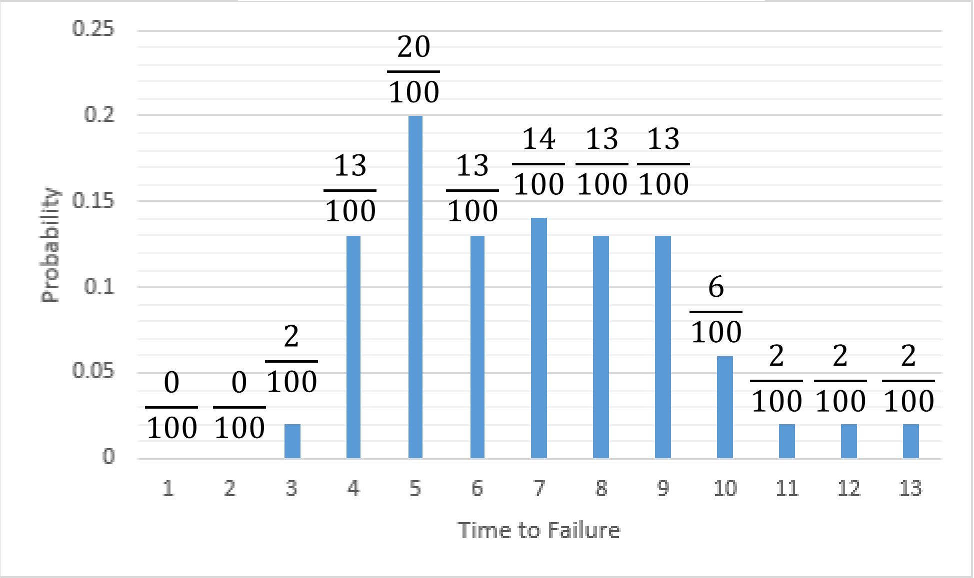
Hazard Function



Probability Density Function

- **Consider the following data for failures by unit time**
- **The ratio of the number of failures between time *[t-1, t)* and the total number of failures can be considered the probability of failing between time *[t-1, t)***
- **The graph represents the probability distribution function (pdf) of failure times**

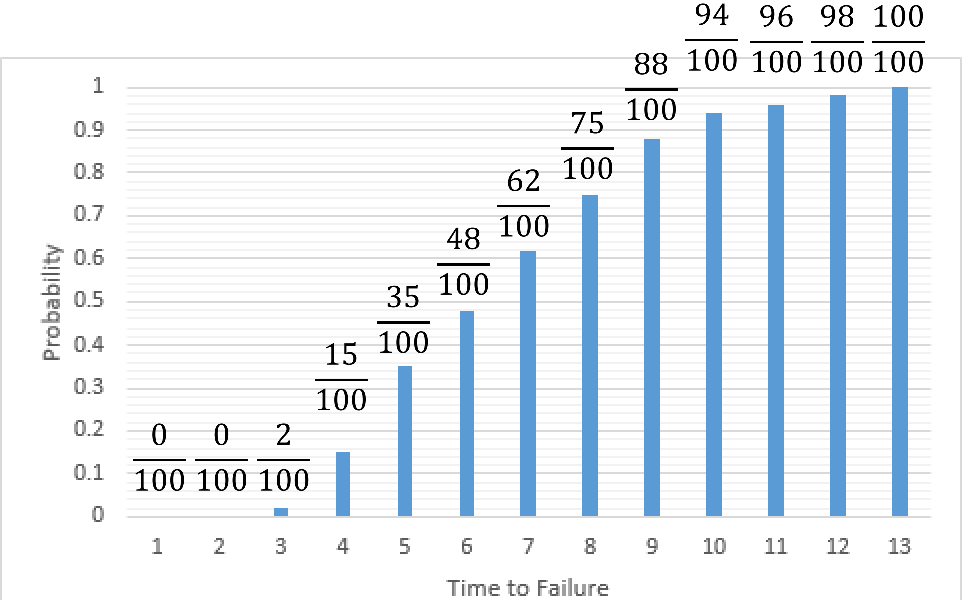
pdf: f(t)		P(t-1 <= T < t)	
t (Time)	Num	Denom	Ratio
1	0	100	0
2	0	100	0
3	2	100	0.02
4	13	100	0.13
5	20	100	0.2
6	13	100	0.13
7	14	100	0.14
8	13	100	0.13
9	13	100	0.13
10	6	100	0.06
11	2	100	0.02
12	2	100	0.02
13	2	100	0.02



Cumulative Distribution Function

- **At each time step we can calculate the cumulative failures just before time *t***
- **The ratio of the number of the cumulative failures before time *t* and the total number of failures at each time step is the Cumulative Distribution Function (CDF), aka *F(t)***
- **The graph represents the Cumulative Distribution Function (CDF) of failure times**

CDF: F(t)		P(T < t)	
t (Time)	Num	Denom	Ratio
1	0	100	0
2	0	100	0
3	2	100	0.02
4	15	100	0.15
5	35	100	0.35
6	48	100	0.48
7	62	100	0.62
8	75	100	0.75
9	88	100	0.88
10	94	100	0.94
11	96	100	0.96
12	98	100	0.98
13	100	100	1

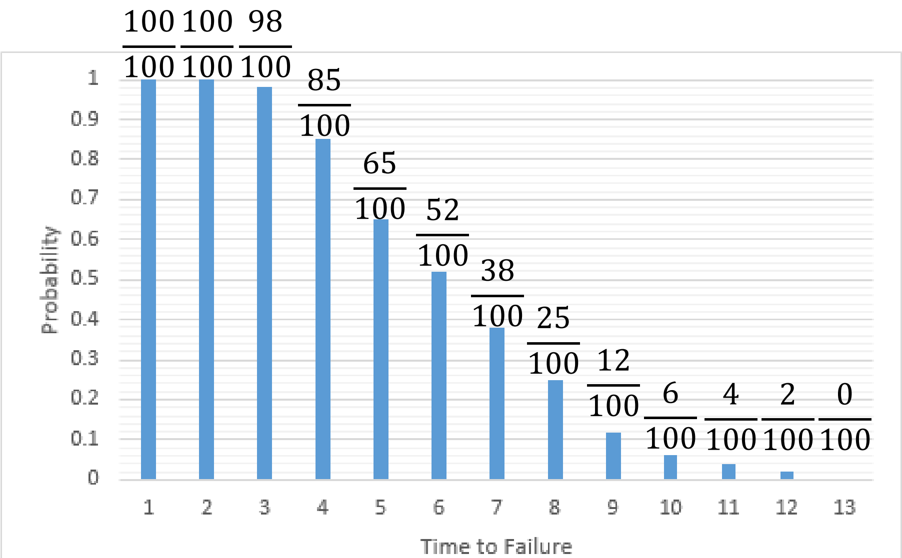


Reliability Function

- The proportion of systems that haven't failed by each time step can be calculated using *F(t)*
  - This is called the reliability function *R(t)* or the survivor function
  - We compute this value by subtracting the number of failures before time *t* from the total number of failures
  - In general this is computed as *1 – F(t)*

CDF: F(t)		P(T < t)	
t (Time)	Num	Denom	Ratio
1	0	100	0
2	0	100	0
3	2	100	0.02
4	15	100	0.15
5	35	100	0.35
6	48	100	0.48
7	62	100	0.62
8	75	100	0.75
9	88	100	0.88
10	94	100	0.94
11	96	100	0.96
12	98	100	0.98
13	100	100	1

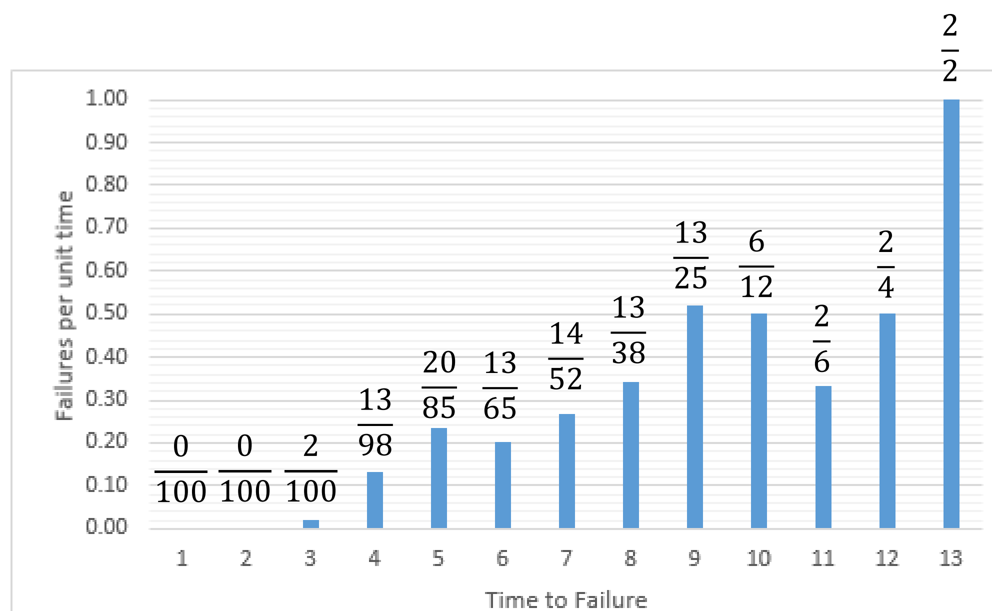
R(t): 1 - F(t)		P(T >=t)	
t (Time)	Num	Denom	Ratio
0	100	100	1
1	100	100	1
2	100	100	1
3	98	100	0.98
4	85	100	0.85
5	65	100	0.65
6	52	100	0.52
7	38	100	0.38
8	25	100	0.25
9	12	100	0.12
10	6	100	0.06
11	4	100	0.04
12	2	100	0.02
13	0	100	0



Hazard Function

- Recall that *f(t)* is the *unconditional* probability that a unit will fail in the interval *(t-1, t)*
- The hazard function *h(t)* is the *conditional* probability that the unit will fail in the interval *(t-1, t)*, given that it has survived until time *t-1*
- The hazard function is computed as $\frac{f(t)}{R(t)}$

h(t)	P(t-1 <= T < t | T >= t-1)		
t (Time)	Num	Denom	Ratio
1	0	100	0.00
2	0	100	0.00
3	2	100	0.02
4	13	98	0.13
5	20	85	0.24
6	13	65	0.20
7	14	52	0.27
8	13	38	0.34
9	13	25	0.52
10	6	12	0.50
11	2	6	0.33
12	2	4	0.50
13	2	2	1.00



Exponential Distribution

- **The exponential distribution is often used as a probability distribution function *f(t)***

$$f(t) = \lambda e^{-\lambda t}$$

- **The CDF *F(t)* is the integral of *f(t)***

$$F(t) = \int_0^t f(t) = \int_0^t \lambda e^{-\lambda t} = 1 - e^{-\lambda t}$$

- **The Reliability Function *R(t)* is *1 – F(t)***

$$\mathrm{R}(t) = 1 - F(t) = 1 - \left(1 - e^{-\lambda t}\right)$$

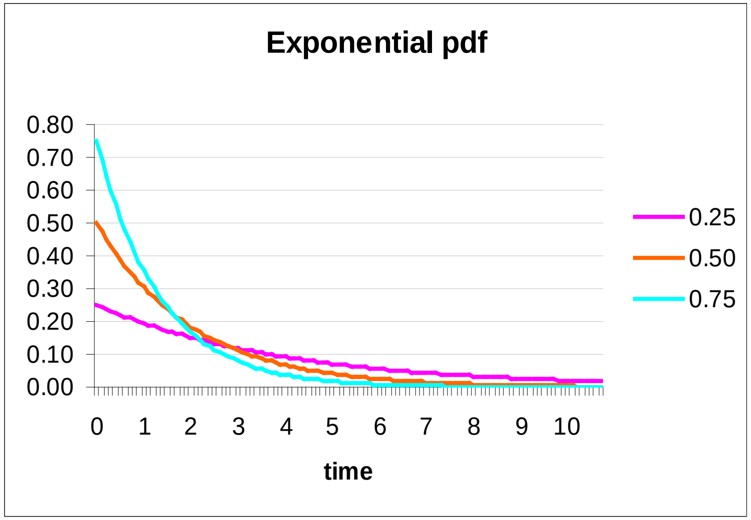
$$\mathrm{R}(t) = e^{-\lambda t}$$

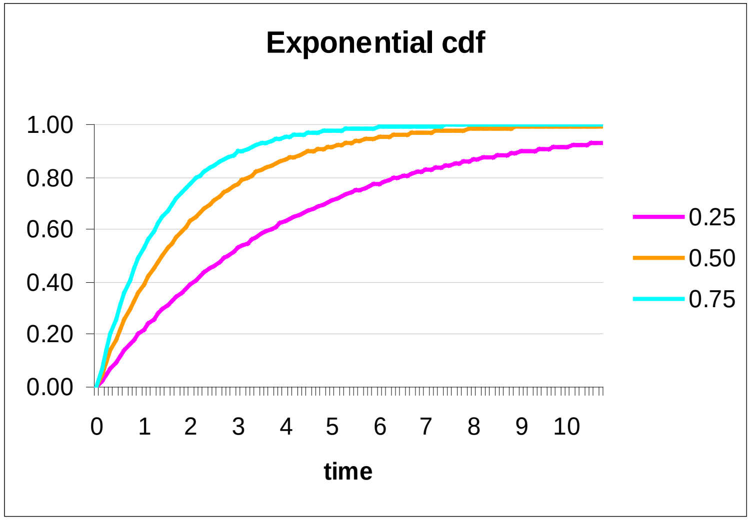
- **The Hazard Function *h(t)* is $\frac{f(t)}{R(t)}$**

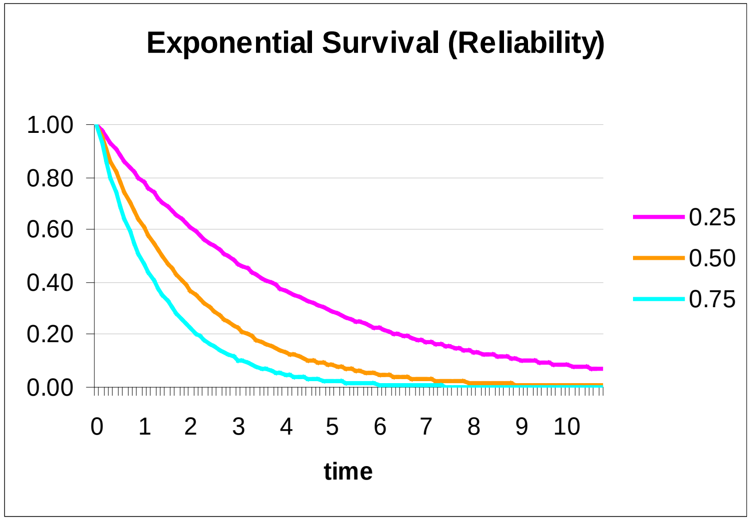
$$\frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

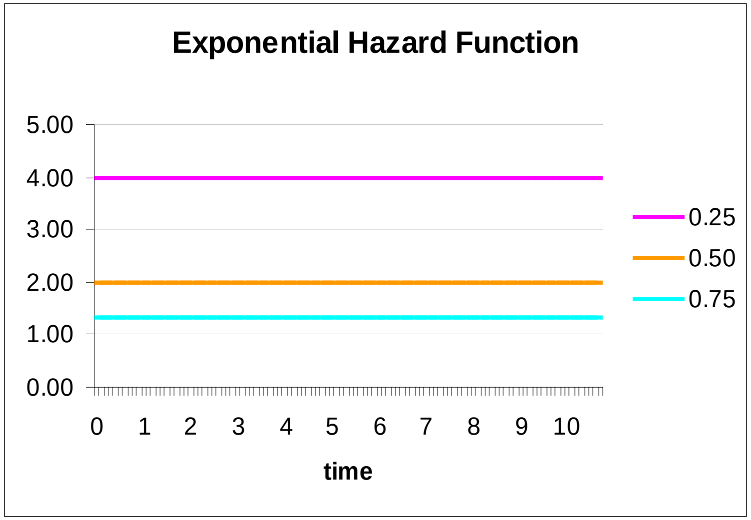
- **For the exponential pdf, the Hazard Function is constant!**

Exponential





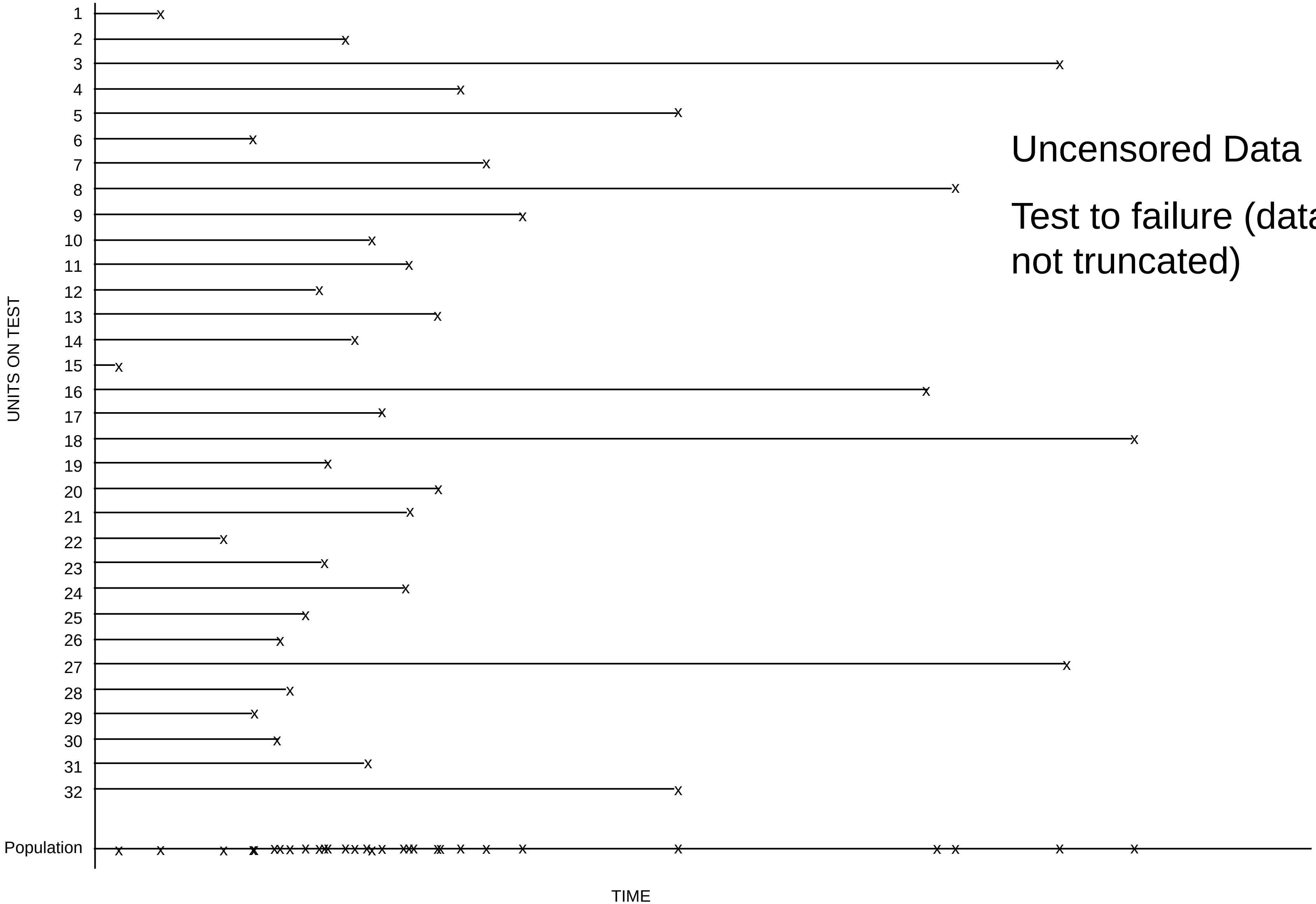




Censored Data

- Reliability data is often censored
- Different types of censoring:
  - Right-censored data – failure times exceed the termination of the test
  - Left-censored data – failure times occur before the first inspection
  - Interval censored data – failure times occur between inspection intervals
  - Random censoring – a component may be damaged during a test

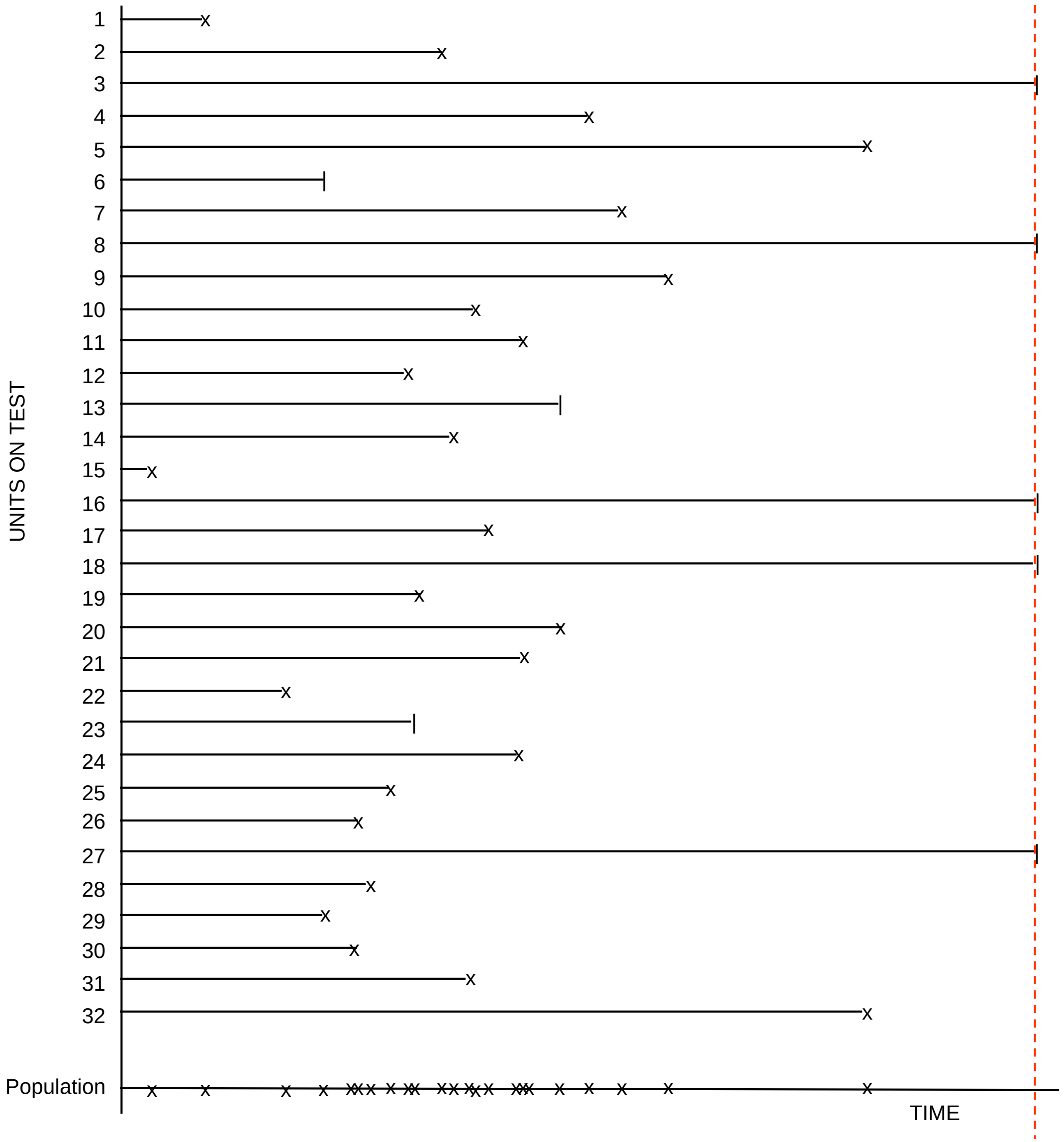
Uncensored Vs. Censored Data



Uncensored Data

Test to failure (data is not truncated)

Uncensored Vs. Censored Data



Censored Data

Test to cut-off time (data is truncated)

Case Study #1

F-22: Infant Mortality

Case Study #1

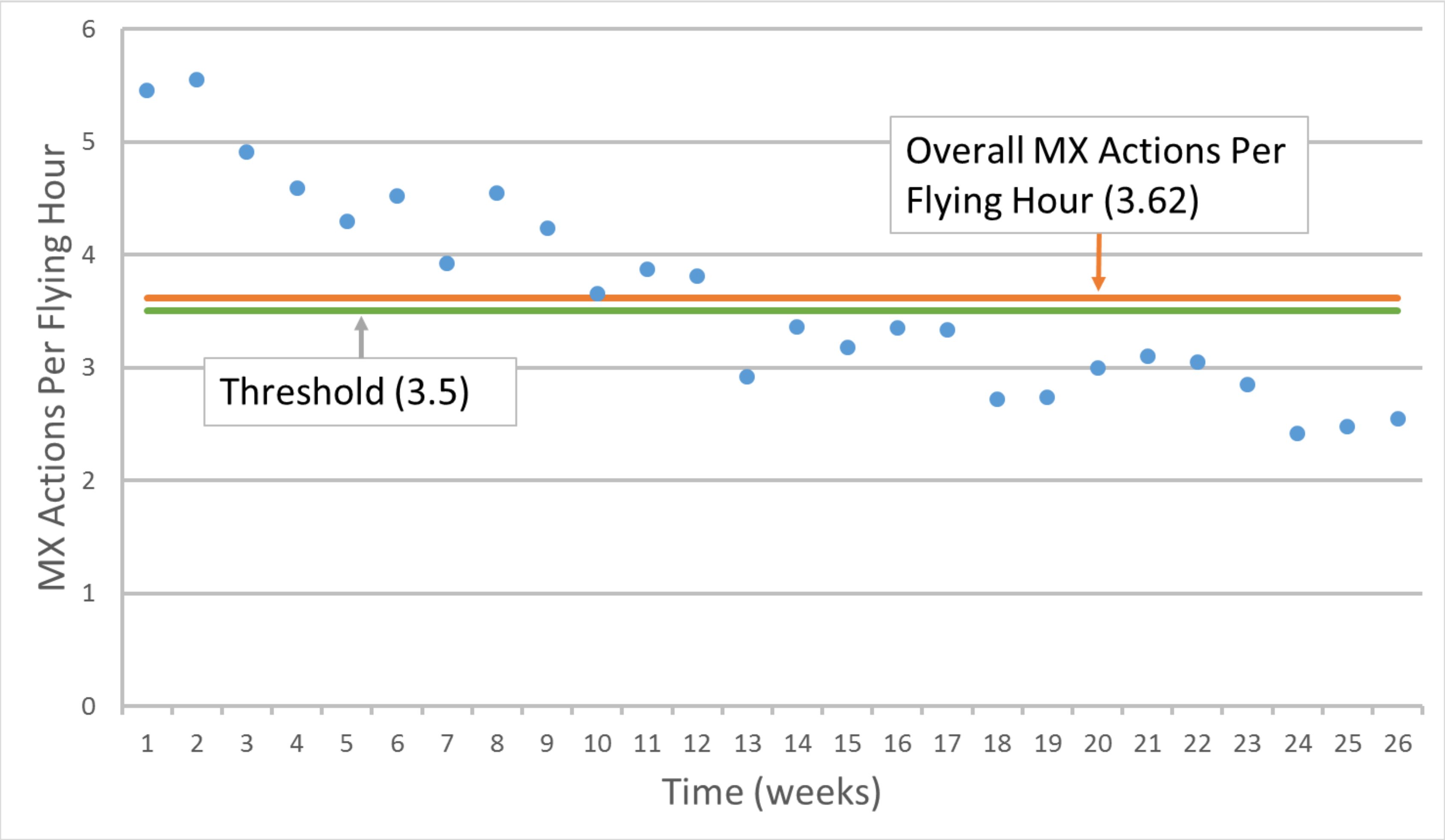
- The system under test was the F-22 Raptor (3 aircraft) from the factory
- Wing utilized aircraft with a constant flying schedule
  - 2-3 sorties/weekday across 3 aircraft
  - Average Sortie Duration = 1.5 hours
- The Initial Operational Test and Evaluation (IOT&E) report provided the following reliability measure:

$$\frac{Total\ MX\ Actions}{Total\ Flying\ Hours} = \frac{2073}{573} = 3.62\ MX\ Actions\ Per\ Flying\ Hour$$

- The measure threshold was 3.5 maintenance (MX) actions per flying hour
- There was anecdotal evidence of initially frequent MX repairs that seemed to diminish over time (6 months)

Case Study #1

- **Post-report analysis revealed additional insights**
- **The high MX actions per flying hour was likely due to infant mortality**



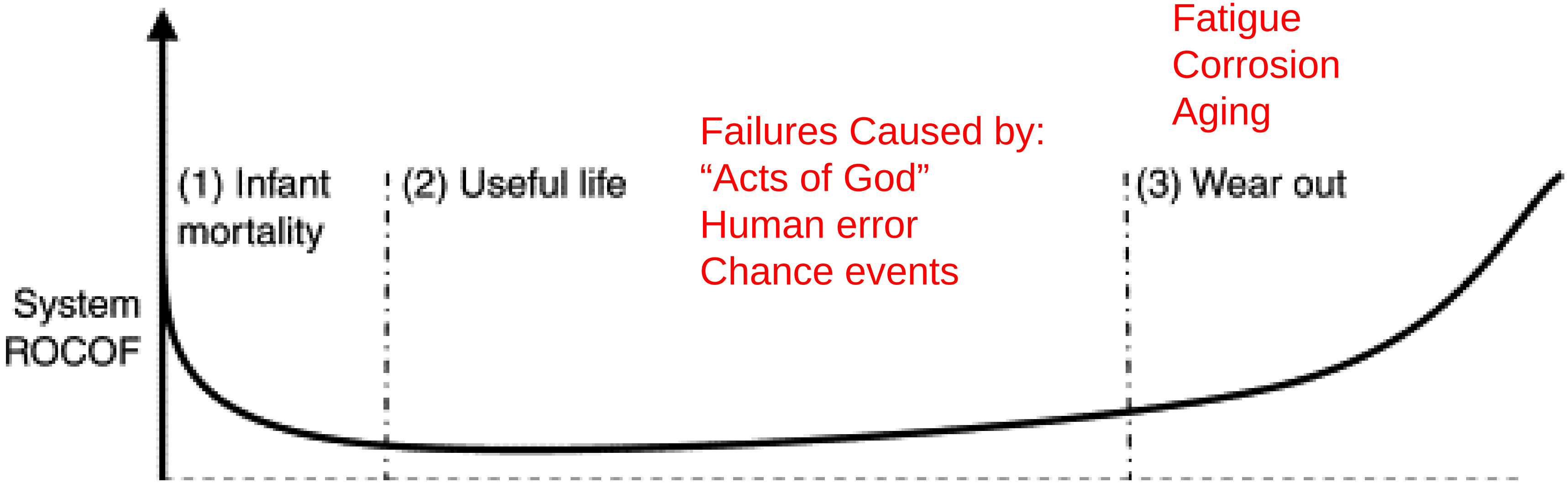
Case Study #1

- **Typical “bathtub” Rate of Occurrence of Failures (ROCOF) curve for repairable systems**

Failures Caused by:
Manufacturing defects
Flaws
Defective parts

Failures Caused by:
Fatigue
Corrosion
Aging

Failures Caused by:
“Acts of God”
Human error
Chance events



Case Study #1

- **Summary**
  - Initial (1-2 months) repair rate of 5-6 MX actions per flying hour
  - At end of test (6 months) repair rate was 2-3 MX actions per flying hour
- **Lessons Learned**
  - Don’t assume the system’s ROCOF curve is in a steady state
  - Look at your data
  - Understand the context of your data

Case Study #2

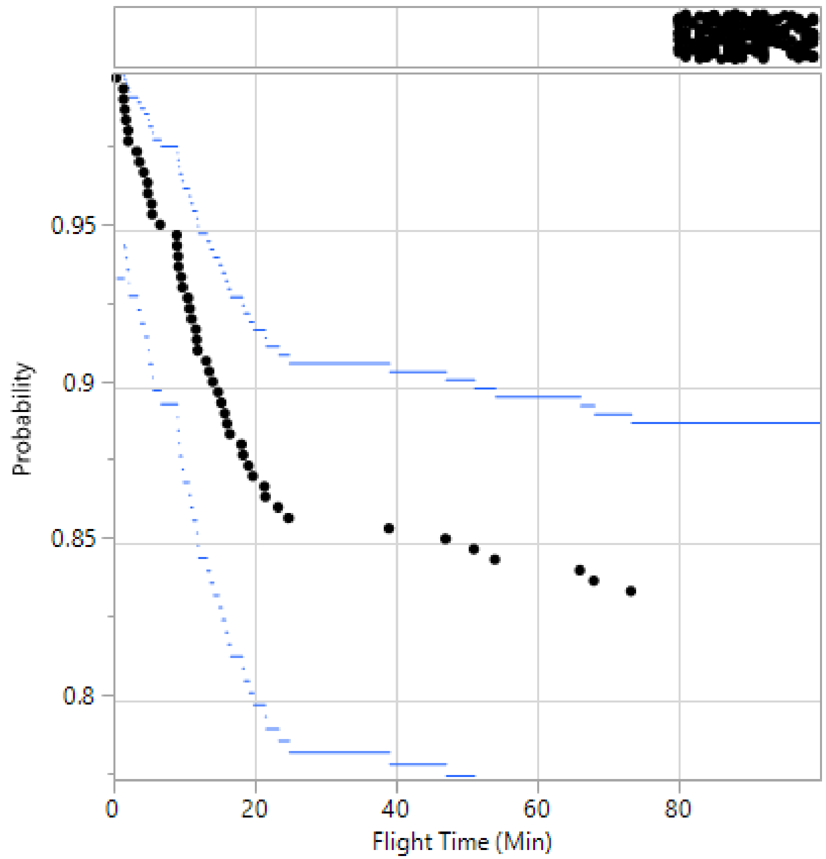
F-22: Weapon System Reliability
(Tell the Whole Story)

Case Study #2

- **Weapon System Reliability (WSR)**
  - Measures the probability that a system will perform satisfactorily for a given mission time when used under specified conditions
- **The F-22 report provided the standard AFPAM 63-128 WSR metric:**
  - Successful Missions / Total Missions
  - WSR was reported as 250/300 = 83.3%
- **Anecdotal stories**
  - If the aircraft had no failures within the first 15 minutes of flight then a successful sortie was very likely
  - The implication was reliability changed over the course of a sortie
  - Indicative of a non-exponential reliability function (non-constant hazard rate)
  - Average sortie duration (ASD) was 1.5 hours

Case Study #2

- Some JMP analysis corroborated the anecdotal evidence
- The majority of failures took place before 20 minutes of flight time



Case Study #2

- **Summary**
  - Standard reliability metric was reported (WSR)
  - Anecdotal evidence led to a more thorough analysis
- **Lessons Learned**
  - Visualize your data
  - Do not assume weapon system reliability is constant over the mission

Case Study #3

RQ-1 Predator: Problems with Assumption of Exponential Distribution

Case Study #3

- A failure density distribution that has a constant failure rate has an exponential reliability distribution
- Many systems exhibit constant failure rates, and the exponential reliability distribution is the simplest to analyze
- Suppose we tested 500 systems for 2000 hours each and observed 100 failures; first calculate the Mean Time Between Failures (MTBF)

$$MTBF = \frac{Total\ Time}{\#\ Failures} = \frac{2000 * 500}{100} = 10,000\ hours$$

- Let the reliability at time *t* be $R(t) = e^{-\lambda t}$ where $\lambda = \frac{1}{MTBF}$
- The reliability at 10,000 hours:

$$R(t) = e^{-\lambda t} = e^{\frac{-1}{10000} * 10000} = 0.37$$

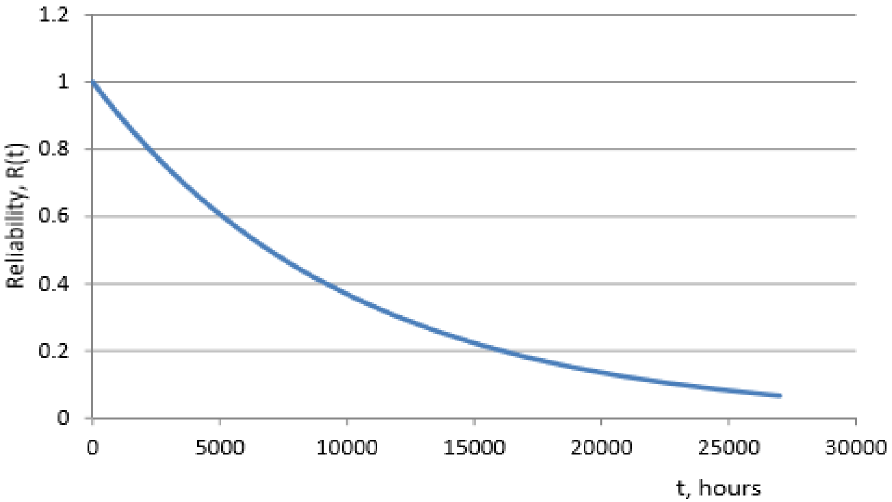


Figure 3. The Exponential Reliability Function whose MTBF is 10,000 hours.

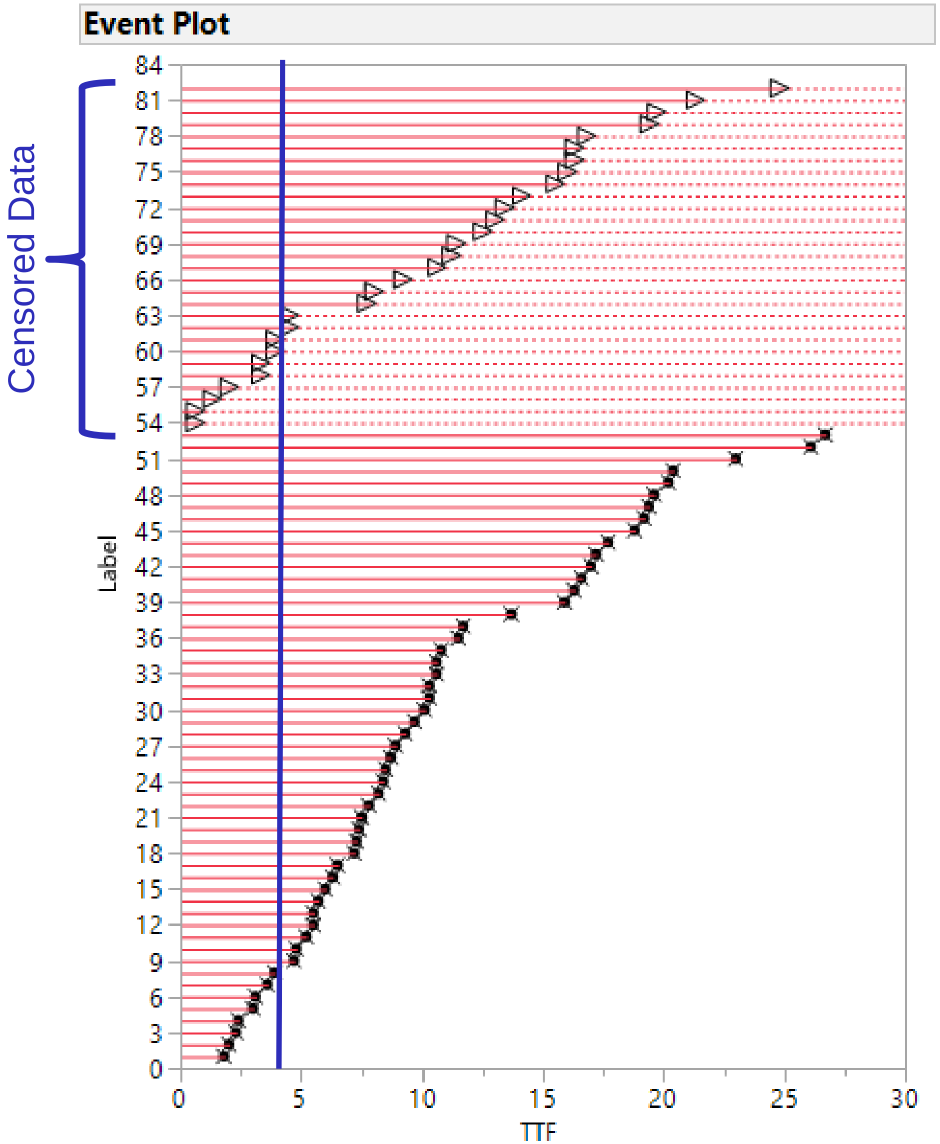
Predator Data

Sortie	Reboot	Reboot	Reboot	Reboot	Reboot	Reboot
1	10.6	14.5	30.0			
2	8.9	18.6	30.0			
3	19.2	24.7	29.4	30.0		
4	5.7	28.7	30.0			
5	17.7	29.4	30.0			
6	6.3	26.7	30.0			
7	7.8	10.9	16.9	19.3	22.3	30.0
8	19.4	30.0				
9	2.0	18.3	25.5	30.0		
10	4.8	13.2	30.0			
11	18.8	30.0				
12	5.2	30.0				
13	5.5	13.7	30.0			
14	13.7	30.0				
15	6.5	14.0	30.0			
16	8.7	30.0				
17	9.3	20.8	30.0			
18	26.7	30.0				
19	10.3	30.0				
20	10.3	18.8	26.1	30.0		
21	10.6	30.0				
22	1.8	22.0	30.0			
23	10.8	28.0	30.0			
24	15.9	30.0				
25	3.6	5.9	25.5	30.0		
26	16.6	30.0				
27	17.0	30.0				
28	26.1	30.0				
29	10.1	17.5	30.0			

- **Test focus: Comm Subsystem**
  - Failure is “Lost Link”
  - Always a reboot
  - Reloaded all parameters
- **Reliability assumed exponential**
  - $MTBF = \frac{\text{Total Time}}{\text{Total Failures}}$
  - $MTBF = \frac{29*30}{53} = 16.42\ hours$
- **Suppose we are concerned with reliability at 4 hours:**
  - $R(t) = e^{-\lambda t} = e^{\frac{-1}{16.42}*4} = 0.78$
- **Problems:**
  - Did not treat data as censored
  - Assumed the exponential reliability function

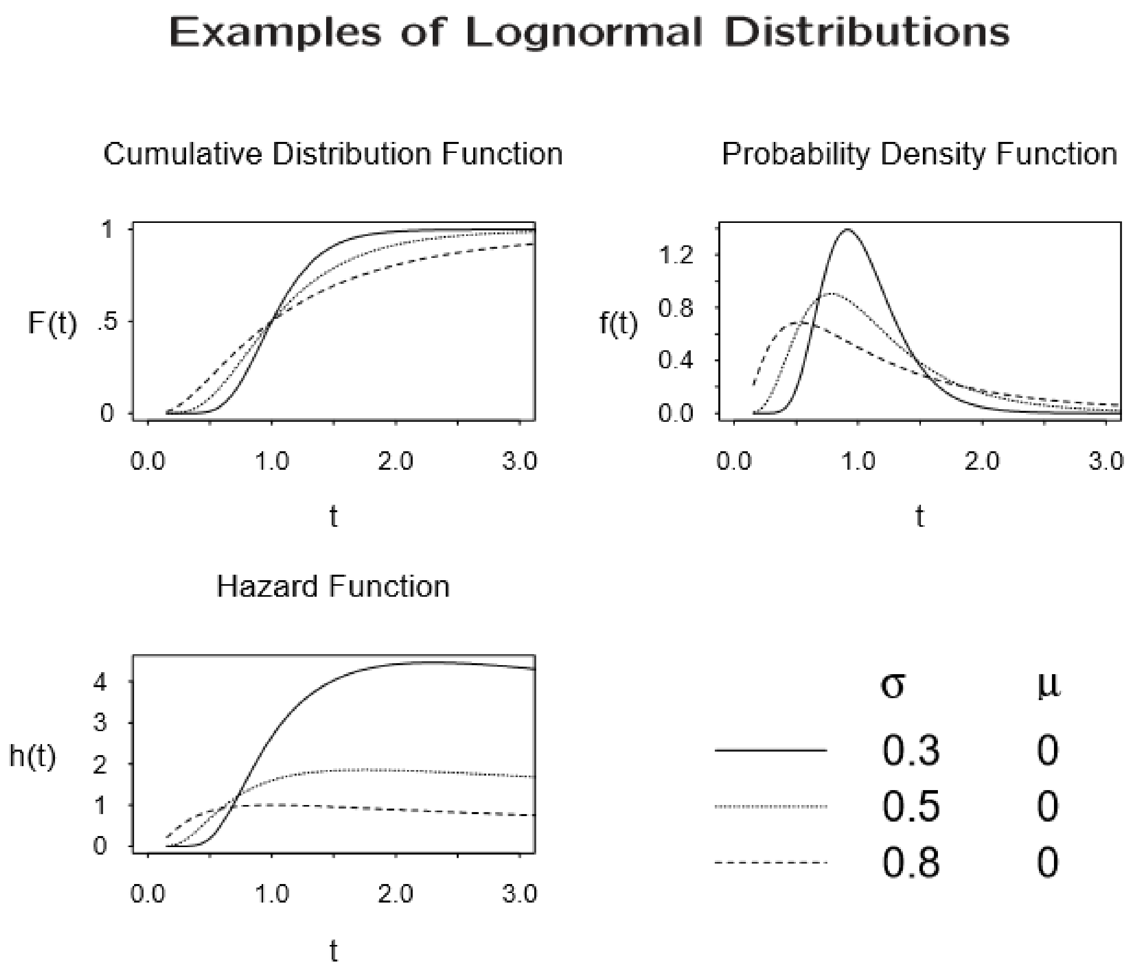
Predator Data

- If we compute the times between failures and account for censoring, the data looks like this
- An “Event Plot” from JMP analysis
- Does it seem reasonable that the reliability at 4 hours is 78% (18/82 failures by 4 hours?)
- What other distributions can be used to model the reliability function?



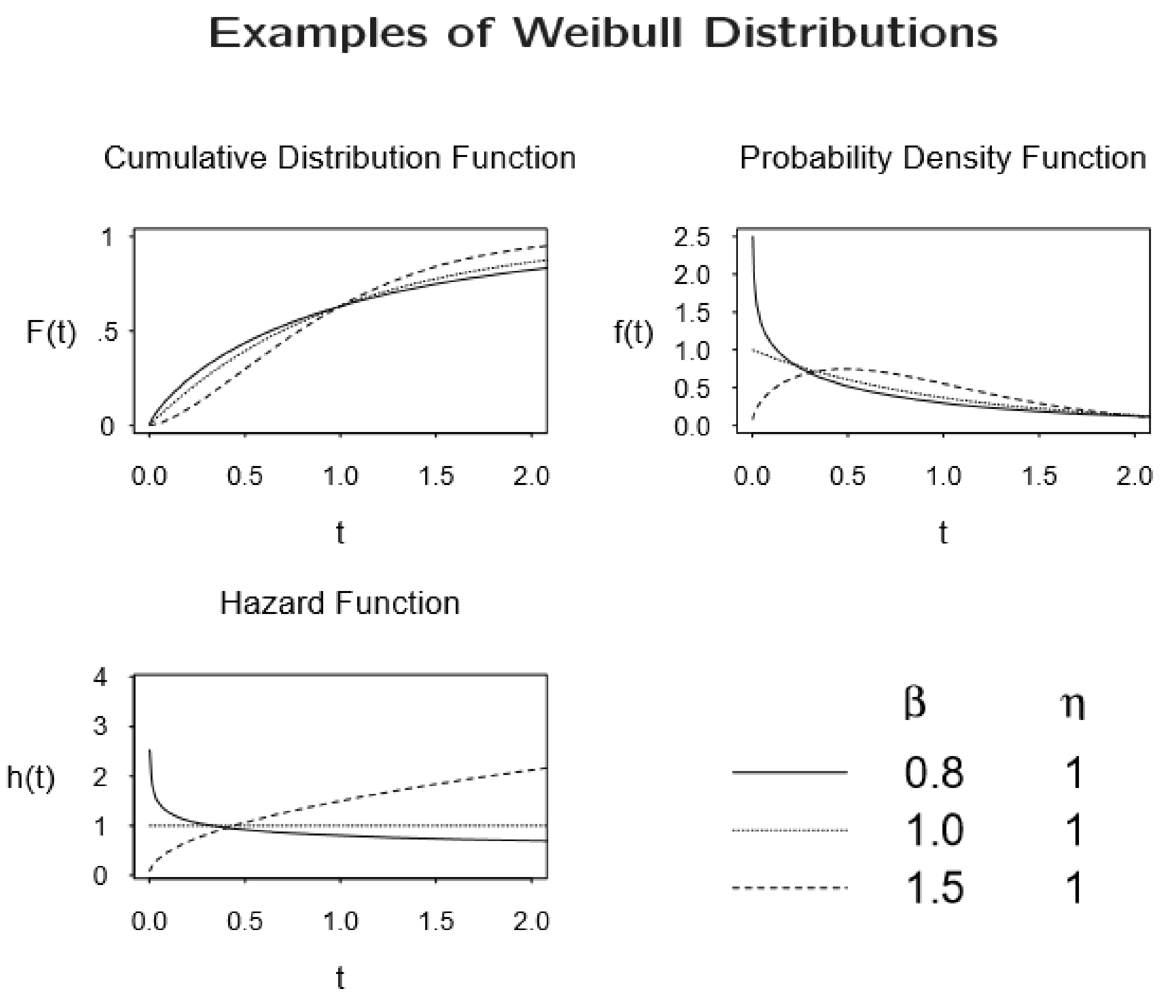
Lognormal Distribution

- **The lognormal distribution is a common model for failure times**



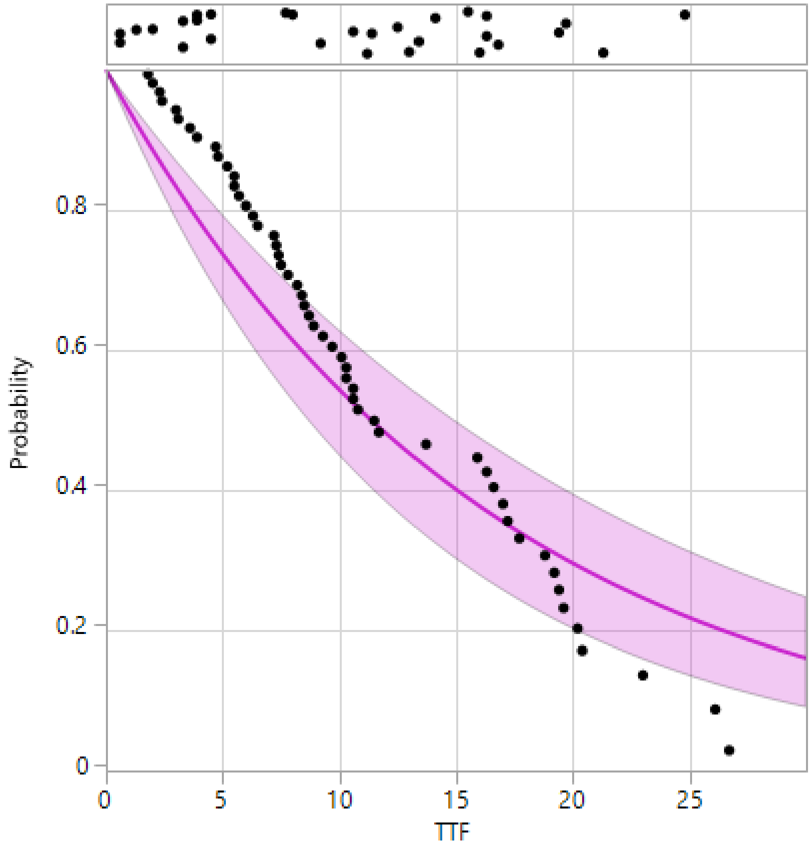
Weibull Distribution

- **The Weibull distribution can be used to model failure-time data with a decreasing or an increasing hazard function**



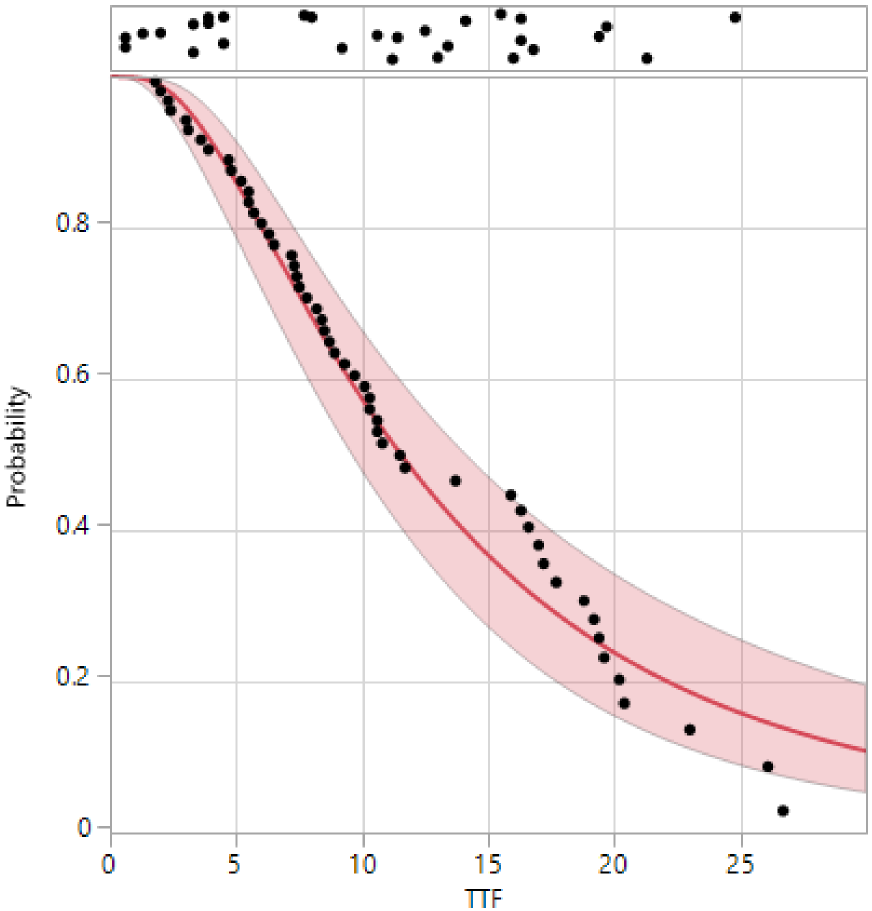
Case Study #3

- **JMP lets us compare the fit of various reliability distributions to our failure time data**
- **This is the fit for an exponential distribution with $\lambda = 16.42$**

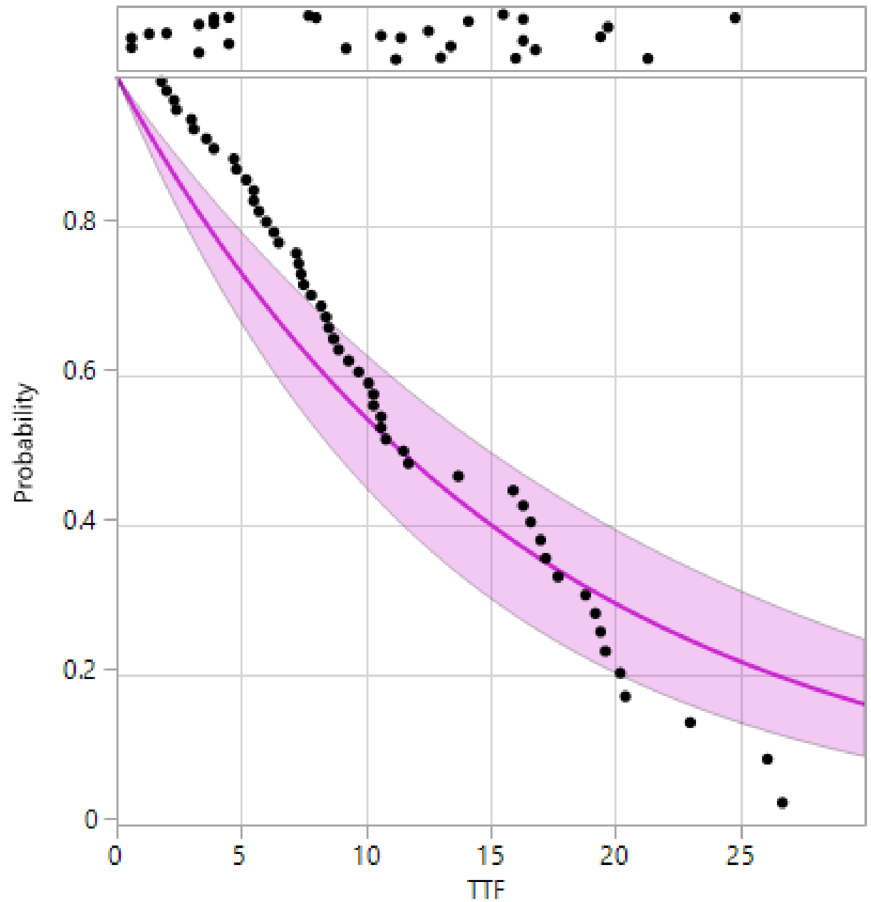


Case Study #3

- **On the left is the fit for a lognormal distribution with $\mu = 2.446$ and $\sigma^2 = 0.773$**



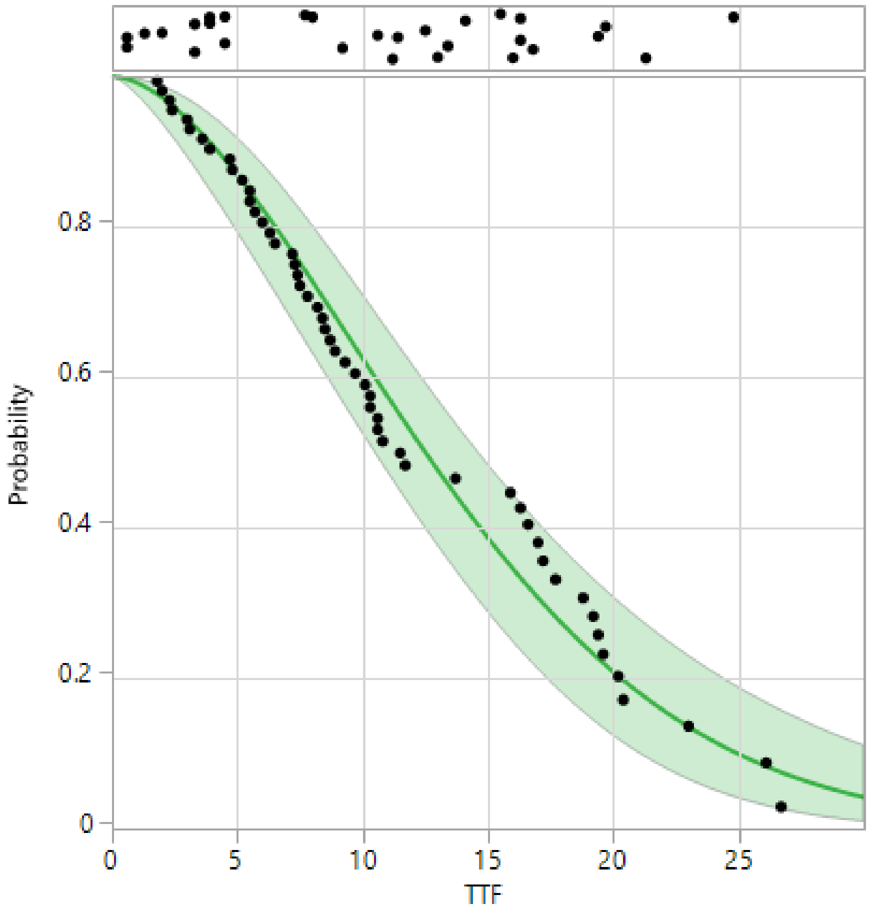
Lognormal Distribution



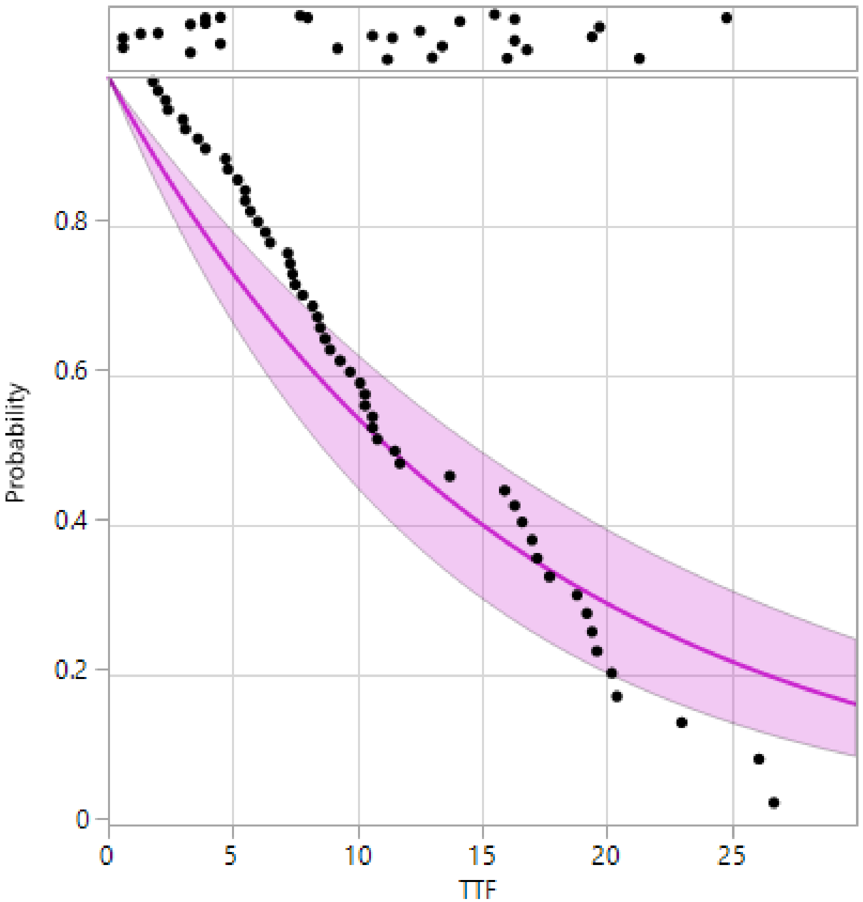
Exponential Distribution

Case Study #3

- **On the left is the fit for a Weibull distribution with $\alpha = 15.448$ and $\beta = 1.738$**



Weibull Distribution



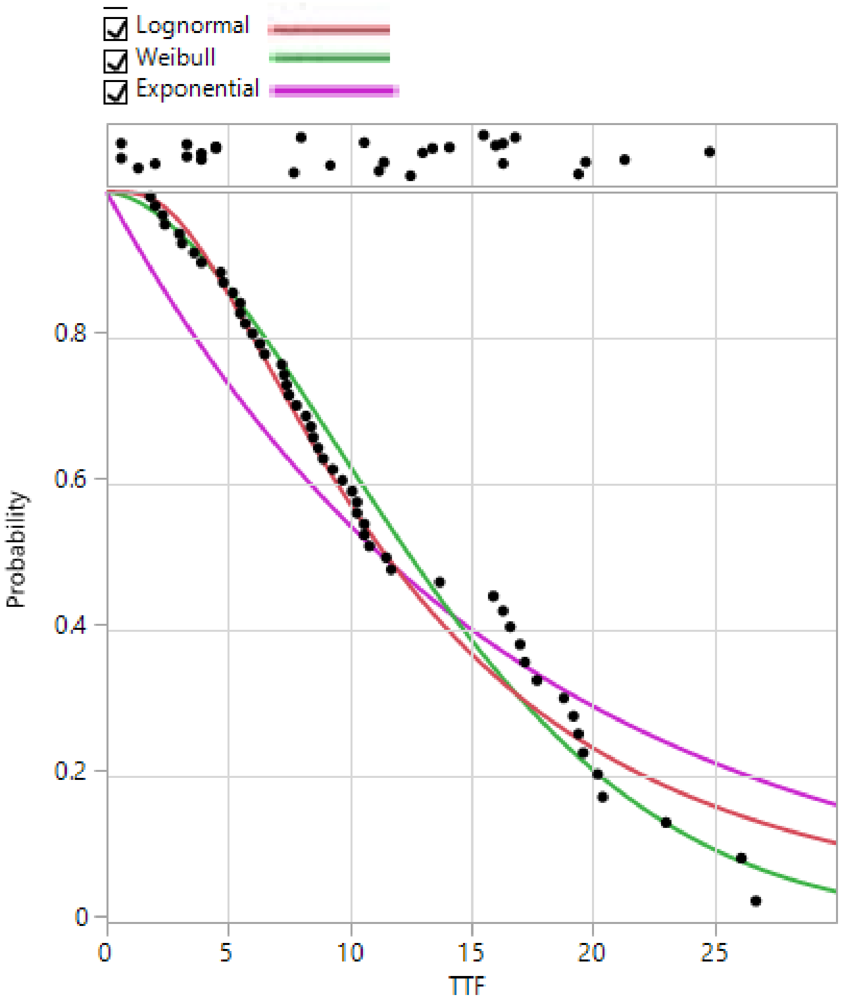
Exponential Distribution

Case Study #3

- **The JMP “Model Comparisons” output shows that the Weibull has the best fit overall**
- **However, which reliability function fits the data best between 0 to 10 hours?**
- **Using the lognormal reliability function, the reliability at 4 hours is 0.915**
- **Recall we computed the reliability at 4 hours to be 0.78 using the exponential function**

Model Comparisons

Distribution	AICc	-2Loglikelihood	BIC
Weibull	386.75918	382.60728	391.42072
Lognormal	388.80777	384.65587	393.46931
Exponential	404.65934	402.60934	407.01606



Case Study #3

- Summary
  - The assumption that RQ-1 Predator failures had an Exponential reliability function was incorrect
  - The Lognormal distribution was a much better model for the reliability curve
- Lessons Learned
  - Visualize your data
  - Consider multiple models for reliability curves

Questions