Test Planning for Observational Studies

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Overview

- Background on Observational Studies
- Methodology for Test Planning
- Poisson Processes
- Examples

Background

- Observational Study – A study (test) conducted where the independent variables (factors) are not under the control of the researcher (test team)
- The primary reason AFOTEC uses observational studies is because the system under test (SUT) is already operational and factor levels cannot be deliberately changed
  - Changing levels may be infeasible
  - Changing levels may adversely affect the operational environment
- Observational studies are common for enterprise software systems:
  - Defense Enterprise Accounting and Management System (DEAMS)
  - Air Force Integrated Personnel and Pay System (AFIPPS)

Background

- Although factor levels cannot be controlled for the purposes of the test, a test matrix that defines the operational battlespace is still useful for test planning
  - The test design matrix is used to determine where and when to send the test observers to observe and gather data
  - The test team attempts to observe as many factor level combinations as possible during the test event
- This is a portion of the test design matrix for the Air Force Integrated Personnel and Pay System (AFIPPS)

Design Point	Business Process	User Role	Affected Member Component	Recipient Classification	Level of Input
1	BP00180.1 Record Disciplinary Actions	HR Specialist	Regular Air Force	Cadet	Base
2	BP00180.1 Record Disciplinary Actions	HR Specialist	Regular Air Force	Enlisted	Base
3	BP00180.1 Record Disciplinary Actions	HR Specialist	Regular Air Force	Officer	Base
4	BP00153.1 - Process Retirement	HR Specialist	Air Force Reserve	Enlisted	ARPC
5	BP00153.1 - Process Retirement	HR Specialist	Air Force Reserve	Officer	ARPC
6	BP00970.1 Manage Leave Requests	Approval Authority	Regular Air Force	Cadet	Unit
7	BP00970.1 Manage Leave Requests	Approval Authority	Regular Air Force	Enlisted	Unit
8	BP00970.1 Manage Leave Requests	Approval Authority	Regular Air Force	Officer	Unit

Problem Statement

- This test design matrix shows the factor labels across the top row
- Each column contains the levels of a factor
- The test will be conducted as an observational study, as any business practices conducted in AFIPPS will affect actual personnel records and human resource (HR) databases

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3	BP00180.1 Record Disciplinary Actions	HR Specialist	Regular Air Force	Officer	Base
4	BP00153.1 - Process Retirement	HR Specialist	Air Force Reserve	Enlisted	ARPC
5	BP00153.1 - Process Retirement	HR Specialist	Air Force Reserve	Officer	ARPC
6	BP00970.1 Manage Leave Requests	Approval Authority	Regular Air Force	Cadet	Unit
7	BP00970.1 Manage Leave Requests	Approval Authority	Regular Air Force	Enlisted	Unit
8	BP00970.1 Manage Leave Requests	Approval Authority	Regular Air Force	Officer	Unit

- Problem
  - Factor level combinations do not occur with equal frequency
  - The test team needs to know how long to schedule an observational test event in order to observe all factor level combinations

Methodology

- **For every observational test event, estimate the frequency of the factor level combinations**

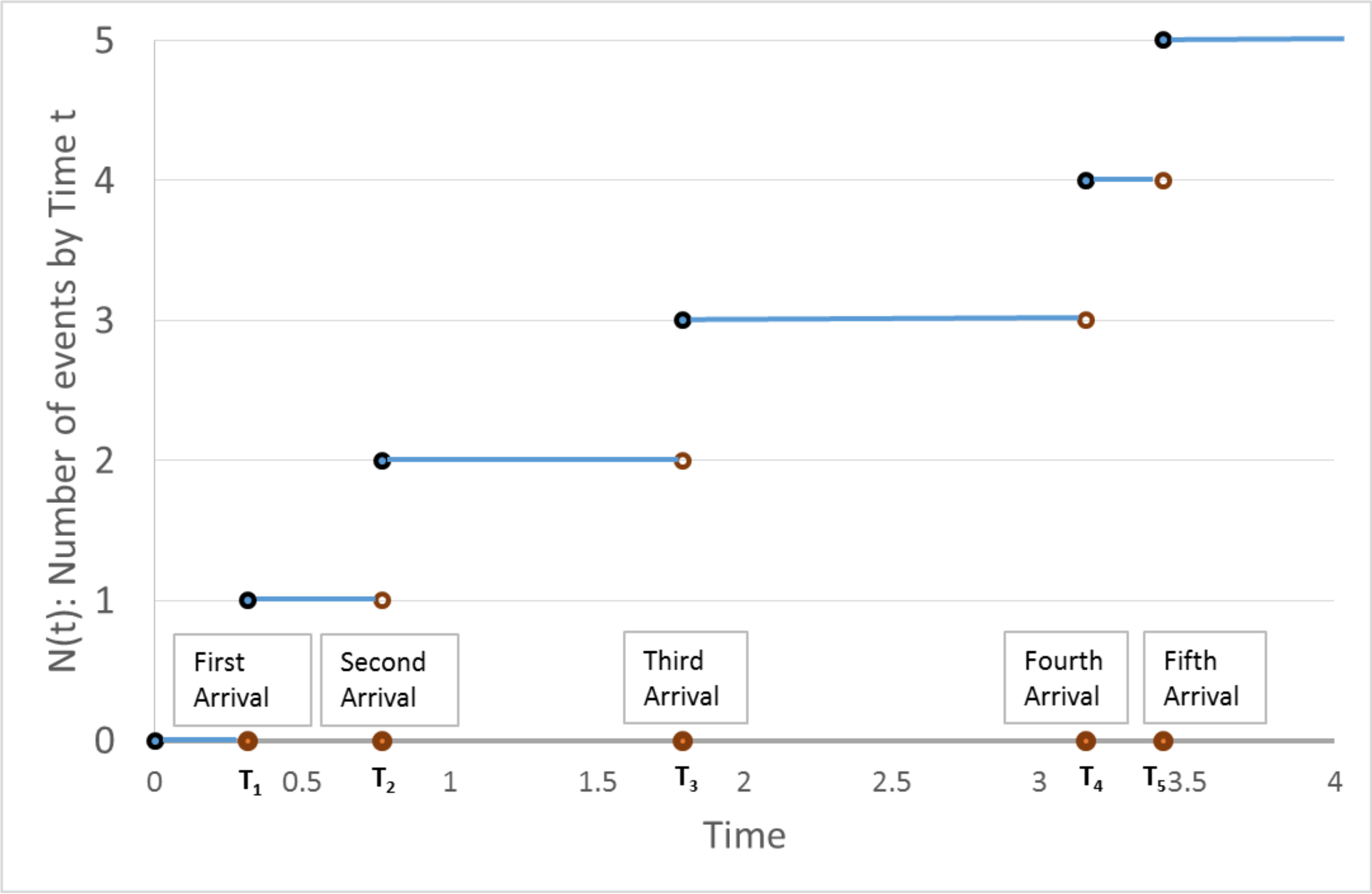
Design Point	Business Process	User Role	Affected Member Component	Recipient Classification	Level of Input	Frequency of Event
1	BP00180.1 Record Disciplinary Actions	HR Specialist	Regular Air Force	Cadet	Base	1 Per 160 Hrs
2	BP00180.1 Record Disciplinary Actions	HR Specialist	Regular Air Force	Enlisted	Base	3 Per 40 Hrs
3	BP00180.1 Record Disciplinary Actions	HR Specialist	Regular Air Force	Officer	Base	3 Per 40 Hrs
4	BP00153.1 - Process Retirement	HR Specialist	Air Force Reserve	Enlisted	ARPC	2 Per 40 Hrs
5	BP00153.1 - Process Retirement	HR Specialist	Air Force Reserve	Officer	ARPC	2 Per 40 Hrs
6	BP00970.1 Manage Leave Requests	Approval Authority	Regular Air Force	Cadet	Unit	3 Per 40 Hrs
7	BP00970.1 Manage Leave Requests	Approval Authority	Regular Air Force	Enlisted	Unit	150 Per 40 Hrs
8	BP00970.1 Manage Leave Requests	Approval Authority	Regular Air Force	Officer	Unit	100 Per 40 Hrs

- **Use a Poisson Process model to determine the time required to observe the rare events**

Siméon Poisson
1781-1840

The Poisson Process

- **The Poisson Process is a common model for scenarios where we are counting the occurrences of certain events that appear to happen at a certain rate, but completely at random**



The Poisson Process

- **Examples of scenarios modeled by Poisson Processes:**
  - **Arrivals of customers at a store**
  - **The number of car accidents in a certain area**
- **Poisson Processes have a rate parameter $\lambda > 0$ that describes the expected number of events to occur in some interval of time**
  - **The number of events in any interval of length *t* is a Poisson random variable with parameter (or mean) λt**
  - **Example: If 5 events happen per hour on average, then $\lambda = 5$, and the number of events occurring in *2* hours is a Poisson random variable with mean $= \lambda t = (5)(2) = 10$**
- **There are a few assumptions:**
  - **The events occur at a certain rate but completely at random within an interval**
  - **The number of events in some interval of length *t* is independent of the number of events in another (disjoint) interval of length *t***

The Poisson Process

- We can use a Poisson Process model to determine the probability that an event will occur by time *t*
- Let $N(t)$ be a Poisson Process with rate λ
- Let T_1 be the time of the first occurrence of a factor level combination we want to observe in an observational test
- Then $P(T_1 > t) = e^{-\lambda t}$, i.e. T_1~Exponential(λ)
- We can use this model to determine the time, *t*, required for a test, given that we want a *Y%* chance to see a rare factor level combination
- A Y% chance to see an event implies $P(T_1 > t) = 1 - \left(\frac{Y}{100}\right)$

$$t = \frac{-ln\left(1 - \left(\frac{Y}{100}\right)\right)}{\lambda}$$

Example 1

- **Suppose a factor level combination occurs randomly 1 time every 3 working days (on average) while a system is operating**
- **We want to determine how long to conduct an observational study so that we have an 80% chance of observing the event**
- **First determine λ (Assume an 8 hour working day)**

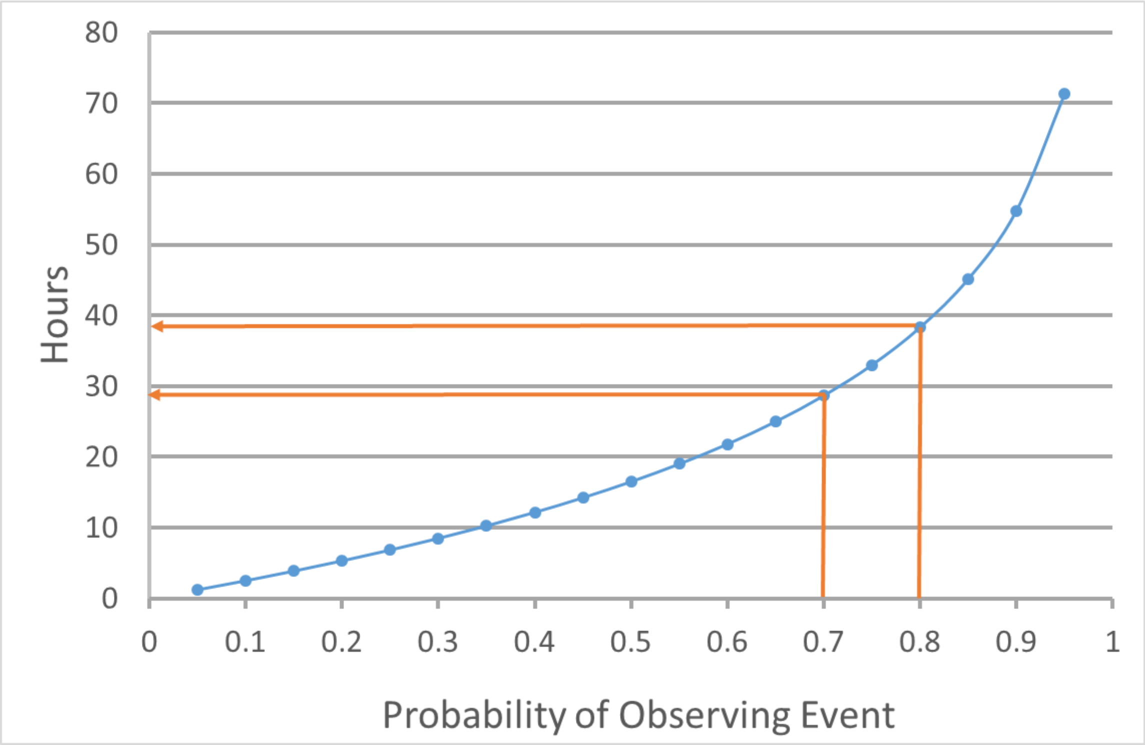
$$\frac{\mathbf{1}\ \boldsymbol{event}}{\mathbf{24}\ \boldsymbol{hours}} \Rightarrow \lambda = \mathbf{0.042}\ \boldsymbol{events/hour}$$

- **Next calculate *t*, the required test time**

$$\boldsymbol{t} = \frac{-\boldsymbol{ln}\left(\mathbf{1} - \left(\frac{\mathbf{80}}{\mathbf{100}}\right)\right)}{\mathbf{0.042}} = \mathbf{38.3}\ \boldsymbol{hours}$$

Example 1

- It is possible to do sensitivity analysis on the probability requirement
- If we only require a 70% probability of observing the event, we can cut nearly 10 hours off the test time



Example 2

- **Suppose we want to observe 3 occurrences of a factor level combination**
- **The interarrival times between events in a Poisson Process are Exponential(λ) random variables (RV); therefore the arrival time of the 3rd event (T_3) is the sum of 3 Exponential(λ) RVs**
- **Hence T_3 is distributed Erlang(3, λ)**
- **The probability that $T_3 > t$ is:**

$$P(T_3 > t) = \sum_{n=0}^{3-1} \frac{1}{n!} e^{-\lambda t} (\lambda t)^n$$

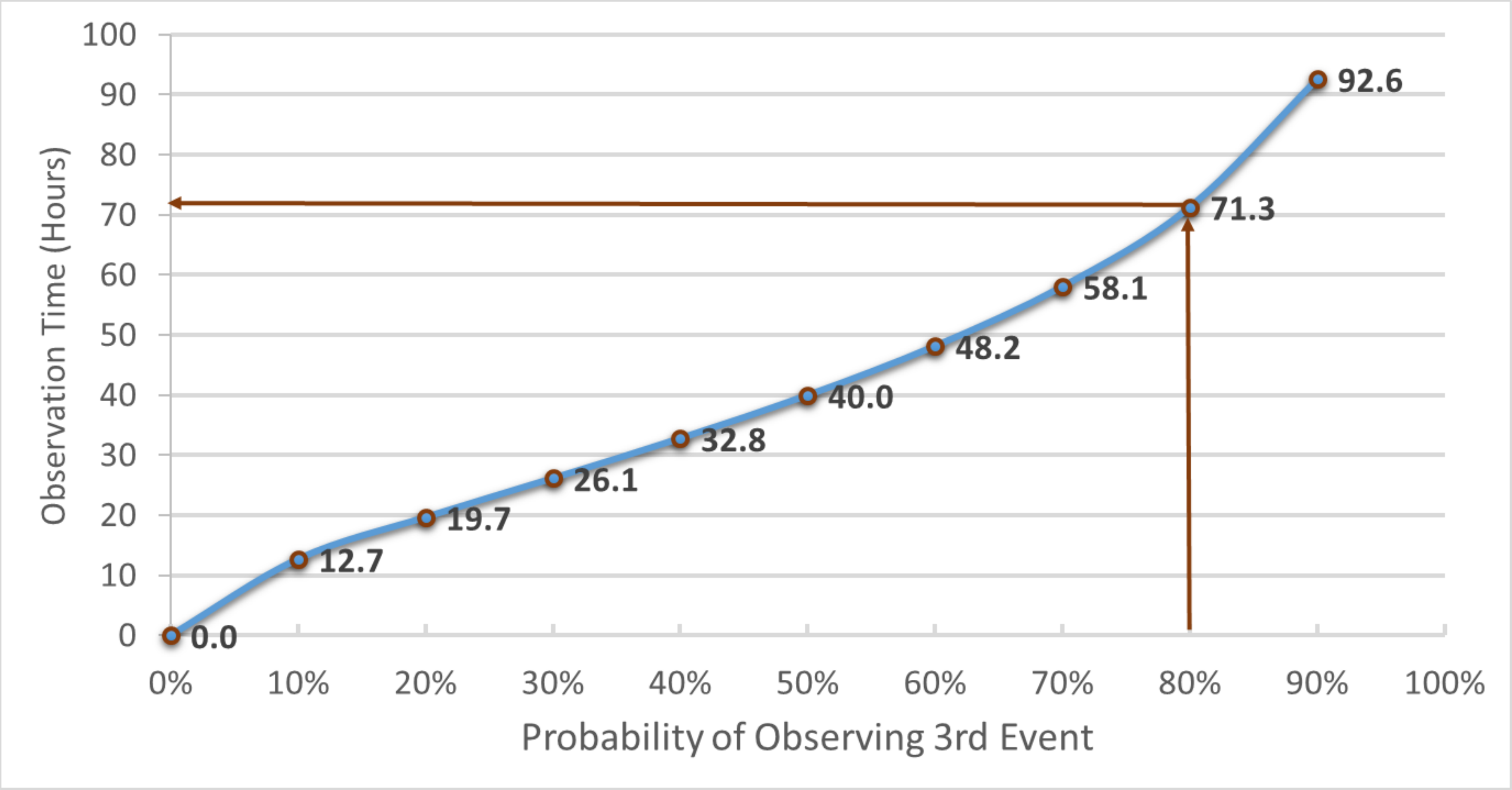
- **Therefore the probability that $T_3 > t$ when $\lambda = 0.042$ is:**

$$P(T_3 > t) = \frac{1}{0!} e^{-\lambda t} (\lambda t)^0 + \frac{1}{1!} e^{-\lambda t} (\lambda t)^1 + \frac{1}{2!} e^{-\lambda t} (\lambda t)^2$$

$$P(T_3 > t) = e^{-0.042t} + e^{-0.042t}(0.042t) + \frac{1}{2} e^{-0.042t}(0.042t)^2$$

Example 2

- The observation time (in hours) required to observe 3 events was calculated for various probabilities (0%, 10%, 20%,...,90%)
- 71.3 hours are required for an 80% chance of observing 3 events



Example 3

- Suppose we have 10 working days to conduct our observational study; The Poisson Process model can be used to determine the probability of seeing a rare factor level combination

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3	BP00180.1 Record Disciplinary Actions	HR Specialist	Regular Air Force	Officer	Base	3 Per 40 Hrs

- There are 80 hours in the test; Now $P(T_1 < t) = 1 - e^{-\lambda t}$

$$\frac{1\ event}{160\ hours} \Rightarrow \lambda = 0.00625\ events/hour$$

$$P(T_1 < 80) = 1 - e^{-(0.00625)(80)} = 1 - 0.607 = 0.393$$

- There is a 39.3% chance of observing the rare event

Summary

- Formula to calculate the test time *t* required to have a *Y%* chance of seeing a factor level combination:

$$t = \frac{-ln\left(1 - \left(\frac{Y}{100}\right)\right)}{\lambda}$$

- Formula to calculate the test time *t* required to have a *Y%* chance of seeing *k* occurrences of a factor level combination:

$$1 - \left(\frac{Y}{100}\right) = \sum_{n=0}^{k-1} \frac{1}{n!} e^{-\lambda t} (\lambda t)^n$$

- Formula to calculate the probability of observing an event by time *t*:

$$P(T_1 < t) = 1 - e^{-\lambda t}$$

Questions